

Eigenmodes of elastic vibrations of quaking neutron star encoded in QPOs on light curves of SGR flares

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Abstract

The Newtonian solid-mechanical theory of non-compressional spheroidal and torsional nodeless elastic vibrations in the homogenous crust model of a quaking neutron star is developed and applied to the modal classification of the quasi-periodic oscillations (QPOs) of X-ray luminosity in the aftermath of giant flares in SGR 1806-20 and SGR 1900+14. A brief outline is given of Rayleigh's energy method which is particular efficient when computing the frequency of nodeless elastic spheroidal and torsional shear modes as a function of multipole degree of nodeless vibrations and two input parameters – the natural frequency unit of shear vibrations carrying information about equation of state and fractional depth of peripheral seismogenic layer. In so doing we discover that the dipole overtones of both spheroidal and torsional nodeless vibrations possess the properties of Goldstone soft modes. It is shown that obtained spectral formulae reproduce the early suggested identification of the low-frequency QPOs from the range $30 \leq \nu \leq 200$ Hz as torsional nodeless vibration modes $\nu_{(0t\ell)}$ of multipole degree ℓ in the interval $2 \leq \ell \leq 12$. Based on this identification, which is used to fix the above mentioned input parameters of derived spectral formulae, we compute the frequency spectrum of nodeless spheroidal elastic vibrations $\nu_{(0s\ell)}$. Particular attention is given to the low-frequency QPOs in the data for SGR 1806-20 whose physical origin has been called into question. Our calculations suggest that these unspecified QPOs are due to nodeless dipole torsional and dipole spheroidal elastic shear vibrations: $\nu_{(0t1)} = 18$ Hz and $\nu_{(0s1)} = 26$ Hz.

Keywords: stars: neutron – stars: oscillations – stars.

1 Introduction

The discovery of QPOs of X-ray luminosity in the aftermath of giant flares SGR 1806-20 and SGR 1900+14 (Israel et al 2005; Strohmayer & Watts 2006; Israel 2007; Watts & Strohmayer 2007) with concomitant interpretation of QPOs as caused by quake-induced differentially rotational seismic vibrations has stimulated remarkable developments in the

magnetar asteroseismology (e.g. Piro 2005, Samuelsson & Andersson 2007a, 2007b; Lee 2007; Levin 2007; Watts & Reddy 2007; Sotani et al 2007; Bastrukov et al 2007a and references therein). Following the above interpretation and presuming the dominant role of the elastic restoring force, the focus of most theoretical works is on computing the frequency spectra of odd-parity torsional mode of shear vibrations and less attention is paid to the even parity spheroidal elastic mode. However, from the viewpoint of modern global seismology (Lay & Wallace 1995; Aki & Richards 2003), the spheroidal vibrational mode in a solid star and planet has the same physical significance as the toroidal one in the sense that these two fundamental modes owe their existence to one and the same restoring force (e.g. McDermott, Van Horn, Hansen 1988; Bastrukov, Weber, Podgainsky 1999, Bastrukov et al 2007b). In this light there is a possibility that, by not considering both these modes on an equal footing, we may miss discovering certain essential novelties which are consequences of solid mechanical laws governing seismic vibrations of superdense matter of neutron stars. Adhering to this attitude and continuing the investigations recently reported by Bastrukov et al (2007a), we derive here spectral equations for the frequency of both spheroidal and torsional elastic nodeless vibrations in the solid crust of quaking neutron star and examine what conclusions can be drawn regarding low-frequency QPOs whose physical nature still remain unclear. In Sec.2 by use of the energy variational method we derive spectral formulae for the frequencies of the nodeless spheroidal and torsional elastic vibrations locked in the finite-depth seismogenic layer. Particular attention is given to the dipole spheroidal and torsional vibrations possessing properties of Goldstone's soft modes. In sec.3, the obtained spectral formulae are applied to a modal analysis of available data on the above mentioned QPOs. The obtained results are highlighted in Sec.4.

2 Frequency of nodeless spheroidal and torsional elastic shear vibrations in homogeneous crust

In this paper we follow the line of argument of the standard two-component, core-crust, model of quaking neutron star (Link, Franco, Epstein 2000) in which metal-like material (composed of nuclei dispersed in the sea of relativistic electrons) of the finite-depth seismogenic crust is treated as a highly robust to compressional distortions elastic continuous medium of a uniform density ρ and characterized by constant value of shear modulus μ . Also, the model presumes that the quake-induced seismic vibrations driven by bulk force of pure shear elastic deformations, which are not accompanied by fluctuations in density $\delta\rho = -\rho \nabla_k u_k = 0$, can be modeled equation of Newtonian, non-relativistic, solid mechanics

$$\rho \ddot{u}_i = \nabla_k \sigma_{ik}, \quad \sigma_{ik} = 2\mu u_{ik}, \quad u_{ik} = \frac{1}{2}[\nabla_i u_k + \nabla_k u_i], \quad u_{kk} = \nabla_k u_k = 0. \quad (1)$$

From now on $u_i(\mathbf{r}, t)$ stands for the field of material displacements in the crust of the depth $\Delta R = R - R_c$ with R and R_c being radii of star and core, respectively. The linear relation between tensors of shear elastic stresses σ_{ik} and shear deformations or strains u_{ik} is the Hooke's law of elastic (reversal) shear deformations. whose strength is characterized by shear modulus

In what follows we focus on poorly investigated regime of quasistatic shear vibrations whose mathematical analysis is quite different from vibrations in the regime of standing waves (Bastrukov et al 2007b). In the latter case, the solution of eigenfrequency problem consists in searching for the wave numbers k of standing waves of material displacements \mathbf{u} obeying the vector Helmholtz equation $\nabla^2 \mathbf{u} + k^2 \mathbf{u} = 0$ supplemented by two boundary conditions on the edges of seismogenic zone, that is, on the core-crust interface and the star surface¹. The two fundamental (orthogonal and different in parity) solutions to the vector Helmholtz equation one of which is given by the positive parity poloidal (polar) field and another by negative parity toroidal (axial) field provide a basis for generally accepted Lamb's classification of vibrational modes as spheroidal or *s*-mode (in which the field of displacement is described by poloidal field) and torsional or *t*-mode (in which the field of displacement is described by poloidal field). The substitution of these fundamental solutions in boundary conditions, which are motivated by physical arguments, leads to the coupled system of transcendent dispersion equations for the wave vector k having the form of bilinear combinations of spherical Bessel and Neumann functions with highly involved distribution of nodes along the radial coordinate of the layer of thickness ΔR . It is the roots of these dispersion equations yield the discrete set of the wave numbers k uniquely related to the frequency of oscillations: $\omega = k c_t$. The systematic application of the above method to the case of torsional elastic modes trapped in the homogeneous crust is presented in (Bastrukov et al 2007a) and pointed out that similar procedure holds true for the case of spheroidal vibrational mode in the homogeneous crust model.

The situation is quite different in the case of long wavelength vibrations, that is, when wave vector $k = (2\pi/\lambda) \rightarrow 0$ and, hence, the wavelength $\lambda \rightarrow \infty$. This limit corresponds to regime of quasistatic, substantially nodeless, vibrations in which the fields of oscillating material displacement subject to the vector Laplace equation (Bastrukov et al 2007b)

$$\nabla^2 \mathbf{u}(\mathbf{r}, t) = 0. \quad (2)$$

Understandably, this last equation can be thought of as the long wavelength limit of vector Helmholtz equation describing standing-wave regime of vibrations (Bastrukov et al 2007a; 2007b). Our interest to the regime of nodeless seismic vibrations is motivated by

¹It is appropriate to note that the very problem of vibrational modes and its solutions for the case of standing wave in entire volume of homogeneous elastically deformable solid sphere has first been tackled and solved by Lamb [11]. The application of Lamb's theory to analysis of seismic vibrations of the solid Earth model is extensively discussed in monographs (Jeffreys 1976; Lapwood & Usami 1981).

arguments of works (Samuelsson & Andersson 2007a; 2002b) in which based on equations of general relativity it was found that low-frequency QPOs in the above mentioned SGR's flare can be identified with low- ℓ nodeless torsional elastic vibrations locked in the crust. And it is somewhat surprising that no such problem has hitherto been properly analyzed on the basis of equations of Newtonian, non-relativistic, solid mechanics. To this end, in (Bastrukov et al 2007b) it has been shown for the first time that the problem of computing frequency spectra of both spheroidal and torsional modes of global nodeless vibrations of the solid star can be uniquely solved with aid of the energy method which is due to Rayleigh. In the present paper this method is extended to spheroidal mode of nodeless elastic vibrations which are considered in one line with torsional mode.

The stating point of the energy variational method is the integral equation of the energy balance

$$\frac{\partial}{\partial t} \int \frac{\rho \dot{u}^2}{2} d\mathcal{V} = - \int \sigma_{ik} \dot{u}_{ik} d\mathcal{V} = -2 \int \mu u_{ik} \dot{u}_{ik} d\mathcal{V} \quad (3)$$

which is obtained by scalar multiplication of equation of solid mechanics, 1, with u_i and integration over the volume of seismogenic layer. For our further purpose the field $\mathbf{u}(\mathbf{r}, t)$ can be conveniently represented in the following separable form

$$\mathbf{u}(\mathbf{r}, t) = \mathbf{a}(\mathbf{r}) \alpha(t) \quad (4)$$

where $\mathbf{a}(\mathbf{r})$ is the field of instantaneous (time-independent) displacements obeying, as follows from (2), to equations

$$\nabla^2 \mathbf{a}(\mathbf{r}) = 0, \quad \nabla \cdot \mathbf{a}(\mathbf{r}) = 0 \quad (5)$$

and $\alpha(t)$ stands for the temporal amplitude of vibrations. Inserting (3) in (2) we arrive at equation for $\alpha(t)$ having the form of standard equation of linear oscillations

$$\frac{dE}{dt} = 0, \quad E = \frac{M \dot{\alpha}^2}{2} + \frac{K \alpha^2}{2} \rightarrow \ddot{\alpha} + \omega^2 \alpha = 0, \quad \omega^2 = \frac{K}{M}, \quad (6)$$

$$M = \int \rho a_i a_i d\mathcal{V}, \quad K = 2 \int \mu a_{ik} a_{ik} d\mathcal{V} \quad a_{ik} = \frac{1}{2} [\nabla_i a_k + \nabla_k a_i]. \quad (7)$$

The solenoidal fields of instantaneous material displacements in two fundamental modes of nodeless vibrations – the spheroidal (normally abbreviated as ${}_0s_\ell$) and the toroidal (abbreviated as ${}_0t_\ell$), are determined by two fundamental (orthogonal and different in parity) solutions to the vector Laplace equation of the vector Laplace equation built on the general solution to the scalar Laplace equation $\nabla^2 \chi(\mathbf{r}) = 0$. In spherical coordinates with fixed polar axis, the solution of (5) corresponding to nodeless vibrations in the spheroidal mode, \mathbf{a}_s , is given by the even parity poloidal (polar) vector field and instantaneous displacements in the torsional mode, \mathbf{a}_t , are by the odd parity toroidal (axial) vector

field:

$$\mathbf{a}_s = \nabla \times \nabla \times (\mathbf{r} \chi), \quad \mathbf{a}_t = \nabla \times (\mathbf{r} \chi), \quad (8)$$

$$\chi(\mathbf{r}) = f_\ell(\mathbf{r}) P_\ell(\cos \theta), \quad f_\ell(\mathbf{r}) = [\mathcal{A}_\ell r^\ell + \mathcal{B}_\ell r^{-(\ell+1)}]. \quad (9)$$

Henceforth $P_\ell(\cos \theta)$ stands for the Legendre polynomial of multipole degree ℓ and \mathcal{A}_ℓ and \mathcal{B}_ℓ are the arbitrary constants to be eliminated from boundary conditions on the core-crust interface and on the star surface. Thus, the problem of computing the frequency spectra of both spheroidal and torsional nodeless vibrations is to fix these constants and compute integrals for integral parameters of vibrations, that is, the inertia M and the stiffness K .

2.1 Spheroidal mode

The poloidal field of nodeless instantaneous displacement \mathbf{a}_s in s-mode is irrotational: $\nabla \times \mathbf{a}_s = 0$ (Bastrukov et al 2007b). To specify \mathcal{A}_ℓ and \mathcal{B}_ℓ we adopt on the core-crust interface, $r = R_c$, the condition of impenetrability of seismic perturbation in the core. On the star edge, $r = R$, we impose the condition that the radial velocity of material displacements equals the rate of spheroidal distortions of the star surface

$$u_r|_{r=R_c} = 0, \quad \dot{u}_r|_{r=R} = \dot{R}(t), \quad R(t) = R[1 + \alpha(t) P_\ell(\cos \theta)]. \quad (10)$$

The solution of resultant algebraic equations leads to following values of arbitrary constants

$$\mathcal{A}_\ell = \frac{\mathcal{N}_\ell}{\ell(\ell+1)}, \quad \mathcal{B}_\ell = -\frac{\mathcal{N}_\ell}{\ell(\ell+1)} R_c^{2\ell+1}, \quad \mathcal{N}_\ell = \frac{R^{\ell+3}}{R^{2\ell+1} - R_c^{2\ell+1}}. \quad (11)$$

Tedious but simple calculation of integrals for inertia M and stiffness K , given by (7), with poloidal field \mathbf{a}_s yields

$$M_s(\ell, \lambda) = \frac{4\pi R^5 \rho}{\ell(2\ell+1)(1-\lambda^{2\ell+1})} \left[1 + \frac{\ell}{(\ell+1)} \lambda^{2\ell+1} \right], \quad (12)$$

$$K_s(\ell, \lambda) = 8\pi R^3 \mu \frac{(\ell-1)(1-\lambda^{2\ell-1})}{\ell(1-\lambda^{2\ell+1})^2} \left[1 + \frac{\ell(\ell+2)}{\ell^2-1} \frac{\lambda^{2\ell-1}(1-\lambda^{2\ell+3})}{(1-\lambda^{2\ell-1})} \right], \quad (13)$$

$$\lambda = \frac{R_c}{R} = 1 - h \quad h = \frac{\Delta R}{R}. \quad (14)$$

The fractional frequency of nodeless spheroidal irrotational shear vibrations as a function of multipole degree ℓ is given by

$$\frac{\omega_s^2(\ell)}{\omega_0^2} = \frac{\nu^2({}_0s_\ell)}{\nu_0^2} = \frac{2(2\ell+1)}{(1-\lambda^{2\ell+1})} \left[\frac{(\ell^2-1)(1-\lambda^{2\ell-1}) + \ell(\ell+2)\lambda^{2\ell-1}(1-\lambda^{2\ell+3})}{(\ell+1) + \ell\lambda^{2\ell+1}} \right] \quad (15)$$

$$\omega_0 = \frac{c_t}{R}, \quad c_t = \sqrt{\frac{\mu}{\rho}}, \quad \nu_0 = \frac{\omega_0}{2\pi}, \quad \nu({}_0s_\ell) = \frac{\omega_s(\ell)}{2\pi}. \quad (16)$$

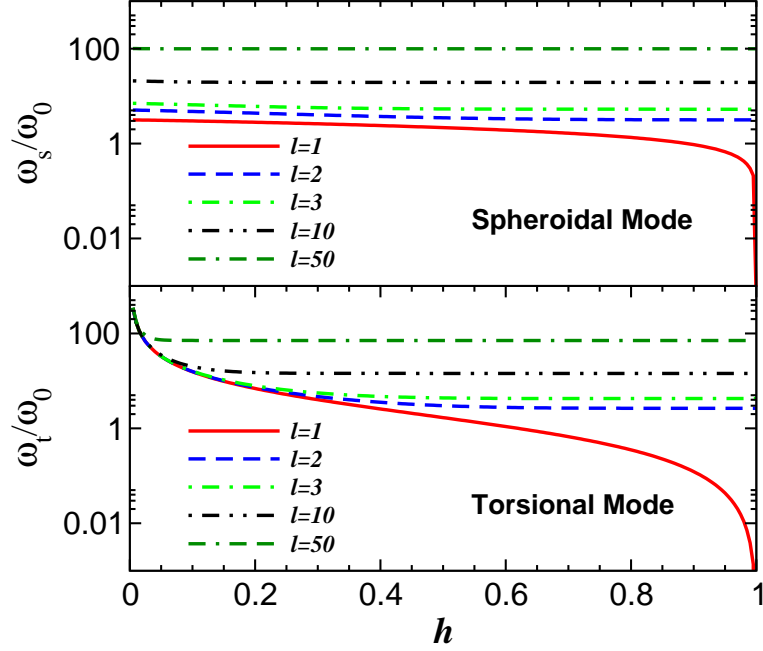


Figure 1: Fractional frequency of nodeless spheroidal and torsional elastic oscillations as a function of depth of seismogenic layer.

It worth noting that in the limit of zero-size radius of the core, $\lambda = (R_c/R) \rightarrow 0$, when entire volume of the star sets in vibrations, we regain the early obtained spectral formula for global nodeless spheroidal nodeless shear vibrations $\nu_{(0s\ell)} = \nu_0 [2(2\ell + 1)(\ell - 1)]^{1/2}$ showing that the lowest overtone of the global nodeless spheroidal oscillations in the entire volume of the star is of quadrupole degree, $\ell = 2$ (Bastrukov et al 2002a 2002b; 2007b). In the meantime, the lowest overtone of spheroidal vibrations trapped in the crust is of the dipole degree, $\ell = 1$. This suggests that the dipole overtone can be considered as a signature of spheroidal vibrations locked in the crust. In the upper panel of Fig.1 we plot the fractional frequency $\omega_s(\ell)/\omega_0$ as a function of $h = \Delta R/R$. This picture indicates that dipole vibration can be thought of as, so called, Goldstone's soft mode whose most conspicuous feature is that the frequency as a function of intrinsic parameter λ of oscillating system $\omega_s(\ell = 1, \lambda) \rightarrow 0$, when $\lambda \rightarrow 0$. In the model under consideration this parameter is given by $\lambda = (R_c/R)$. The limit $\lambda = 0$ belongs to translation displacement of the center-of-mass of the star, not a vibration; this is clearly seen from the equation for energy (Hamiltonian) of harmonic oscillations (6). In the next section we show how the input parameters of obtained spectral equation (15), namely, the natural unit of frequency ν_0 and the depth h of seismogenic layer can be extracted from the data on QPOs for SGS.

2.2 Torsion mode

For the torsional oscillations locked in the crust, the constants \mathcal{A}_ℓ and \mathcal{B}_ℓ are eliminated from the following boundary conditions

$$u_\phi|_{r=R_c} = 0, \quad u_\phi|_{r=R} = [\phi_R \times \mathbf{R}]_\phi, \quad (17)$$

$$\phi_R = \alpha(t) \nabla_{\hat{\mathbf{n}}} P_\ell(\zeta), \quad \nabla_{\hat{\mathbf{n}}} = \left(0, \frac{\partial}{\partial \theta}, \frac{1}{\sin \theta} \frac{\partial}{\partial \phi} \right). \quad (18)$$

First is the no-slip condition on the core-crust interface, $r = R_c$, implying that the amplitude of differentially rotational oscillations is gradually decreasing from the surface to the core. The boundary condition on the star surface, $r = R$, is dictated by symmetry of the general toroidal solution of the vector Laplace equation. The support of this last boundary condition lends further considerations showing that it leads to correct expression for the moment of inertia of a rigidly rotating star. The resultant algebraic equations steaming from above boundary conditions lead to

$$\mathcal{A}_\ell = \mathcal{N}_\ell, \quad \mathcal{B}_\ell = -\mathcal{N}_\ell R_c^{2\ell+1}, \quad \mathcal{N}_\ell = \frac{R^{\ell+2}}{R^{2\ell+1} - R_c^{2\ell+1}}. \quad (19)$$

Tedious calculation of integrals for M_t and K_t leads to

$$M_t = \frac{4\pi\ell(\ell+1)}{(2\ell+1)(2\ell+3)} \frac{\rho R^5}{(1-\lambda^{2\ell+1})^2} \times \left[1 - (2\ell+3)\lambda^{2\ell+1} + \frac{(2\ell+1)^2}{2\ell-1} \lambda^{2\ell+3} - \frac{2\ell+3}{2\ell-1} \lambda^{2(2\ell+1)} \right], \quad (20)$$

$$K_t = \frac{4\pi\ell(\ell^2-1)}{2\ell+1} \frac{\mu R^3}{(1-\lambda^{2\ell+1})} \left[1 + \frac{(\ell+2)}{(\ell-1)} \lambda^{2\ell+1} \right] \quad (21)$$

$$\lambda = \frac{R_c}{R} = 1 - h \quad h = \frac{\Delta R}{R}. \quad (22)$$

In the limit of zero-size radius of the core, $\lambda = (R_c/R) \rightarrow 0$, corresponding to torsional oscillations in the entire volume of the star we regain the early obtained spectral formula for the global nodeless torsional elastic vibrations $\nu({}_0t_\ell) = \nu_0 [(2\ell+3)(2\ell-1)]^{1/2}$ showing that in case of global torsional oscillations the lowest overtone is of quadrupole degree (Bastrukov et al 2002a; 2002b; 2007a; 2007b). However, this is not the case when we consider torsional nodeless oscillations locked in the seismogening layer of finite depth $\Delta R = R - R_c$. For $\ell = 1$, equations (20) and (21) are reduced to

$$M_t(\ell=1, \lambda) = \frac{8\pi\rho R^5}{15(1-\lambda^3)^2} [1 - 5\lambda^3 + 9\lambda^5 - 5\lambda^6], \quad (23)$$

$$K_t(\ell, \lambda) = 8\pi\mu R^3 \frac{\lambda^3}{(1-\lambda^3)}, \quad (24)$$

$$\omega_t^2(\ell=1, \lambda) = \omega_0^2 \frac{15\lambda^3(1-\lambda^3)}{(1-\lambda)^3(1+3\lambda+6\lambda^2+5\lambda^3)} \quad 0 \leq \lambda < 1. \quad (25)$$

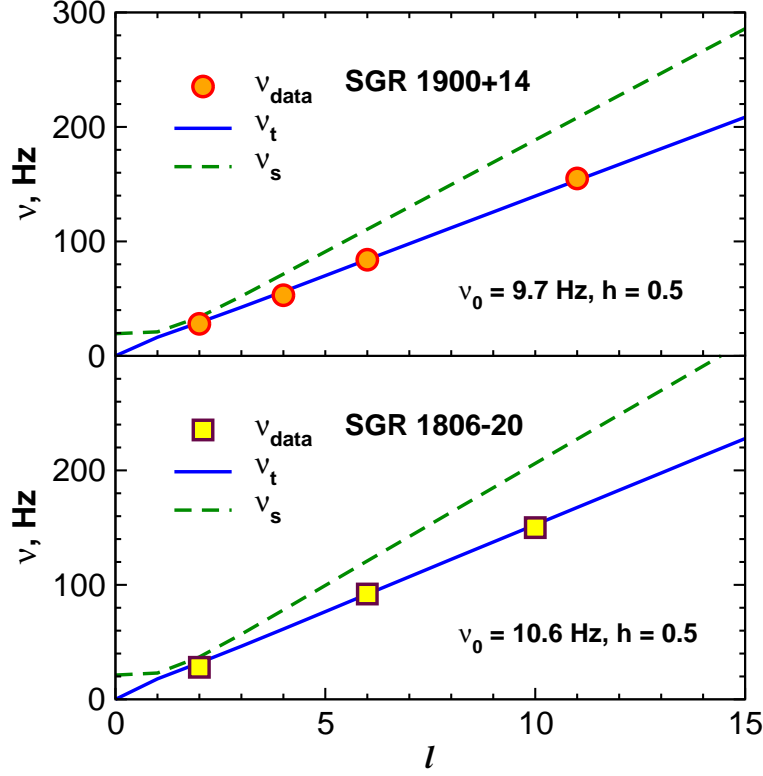


Figure 2: Theoretical curves for the frequency of spheroidal (dashed) and torsional (solid) nodeless elastic oscillations computed with aid of spectral formulae for frequency of spheroidal, eq.(15), and torsional, eqs. (26)-(27), modes as functions of multipole degree in juxtaposition with data (symbols) on QPOs for SGR 1900+14 and for SGR 1806-20. The modal identification is taken from (Samuelsson and Andersson 2007b; Watts & Strohmayer 2007).

In the limit zeroth core, $\lambda \rightarrow 0$, the stiffness $K_t \rightarrow 0$ and the mass parameter getting the form of moment of inertia of absolutely rigid solid star of mass \mathcal{M} and radius R : $M_t(\ell = 1, \lambda = 0) = (2/5)\mathcal{M}R^2$. This consideration again shows that the dipole vibration exhibits features of the Goldstone soft mode owing its emergence to the trapping of torsional shear oscillations in the peripheral crust of finite depth.

The general spectral equation for the fractional frequency of nodeless torsional oscillations of arbitrary multipole degree ℓ , computed with aid of equations (20) and (21), can be presented in the following analytic form

$$\frac{\omega_t^2(\ell)}{\omega_0^2} = \frac{\nu^2(\ell)}{\nu_0^2} = [(\ell + 2)(\ell - 1)] p_\ell(\nu_0, \lambda) \quad (26)$$

$$p_\ell(\nu_0, \lambda) = 4 \left[1 - \frac{1}{2(\ell + 2)} \right] \left[1 + \frac{1}{2(\ell - 1)} \right] (1 - \lambda^{2\ell+1}) \times \left\{ 1 - \frac{\ell - \lambda^{2\ell+1}[(\ell + 2) + (2\ell - 1)(2\ell + 3) - (2\ell + 1)^2\lambda^2 + (2\ell + 3)\lambda^{2\ell+1}]}{(2\ell - 1) - \lambda^{2\ell+1}[(2\ell - 1)(2\ell + 3) - (2\ell + 1)^2\lambda^2 + (2\ell + 3)\lambda^{2\ell+1}]} \right\}. \quad (27)$$

The practical usefulness of such representation is extensively discussed in (Bastrukov et

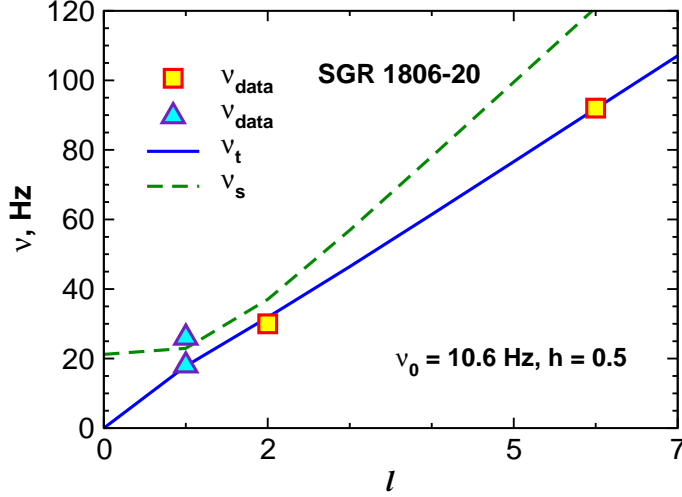


Figure 3: Theoretical predictions for the frequency of spheroidal (dashed) and torsional (solid) nodeless elastic oscillations as a function of multipole degree ℓ in juxtaposition with data (symbols) on QPOs SGR 1806-20. The identification of QPOs pictured by squares is taken from (Samuelsson & Andersson 2007b; Watts & Strohmayer 2007). Based on the results of this latter work, our calculations suggest that low frequency QPOs discovered in (Israel et al 2005), that are pictured by triangles, can be identified as dipole toroidal and dipole spheroidal nodeless vibration, respectively: $\nu_{(0)t_1} = 18$ Hz and $\nu_{(0)s_1} = 26$ Hz.

al 2007a). The fractional frequency as a function of $h = \Delta R/R$ is pictured in down panel of Fig.1 which show that the lowest overtone of torsional vibrations trapped in the crust is of dipole degree and that dipole overtone of differentially rotational vibrations of the crust against core possesses properties of the Goldstone’s soft mode.

3 Application to SGR 1900+14 and SGR 1806-20

The obtained spectral formulae (15) and (26)-(27) describe the frequencies of both spheroidal and torsional nodeless oscillations as functions of the multipole degree ℓ . The natural unit of frequency ν_0 of shear elastic vibrations and the fractional depth h of peripheral seismogenic layer are input parameters carrying information about material properties of neutron star matter (density, shear modulus) and geometrical sizes of star and seismoactive zone. Considering the observation data for SGR 1900+14 and SGR 1806-20, we demonstrate here how the obtained spectral equations can be used to eliminate some uncertainties in identification of QPOs.

First, we examine the agreement of obtained spectral formula (26)-(27) for torsion mode with identification of the QPOs frequencies from interval $30 \leq \nu \leq 200$ Hz with frequencies of nodeless torsional vibrations of multipole degree ℓ from interval $2 \leq \ell \leq 12$

suggested in works (Samuelsson & Andersson 2007a; 2007b). In so doing we use the proposed in these latter works identification of QPOs in SGR 1900+14 data [namely, $\nu_{(0t_2)} = 28$ Hz; $\nu_{(0t_4)} = 53$ Hz; $\nu_{(0t_6)} = 84$ Hz, $\nu_{(0t_{11})} = 155$ Hz borrowed from Table 1 of paper (Samuelsson & Andersson 2007b)] as reference points and vary parameters ν_0 and h entering our spectral formula (15) for torsional mode so as to attain the best fit of these points. The result of this procedure is shown in upper panel of Fig.2 by solid line. Then, making use of the fixed in the above manner parameters ν_0 and h , we compute (with the aid of spectral formula (15)) the frequency of spheroidal mode $\nu_{(0s_\ell)}$. The application of this procedure to modal analysis of QPOs data for SGR 1806-20 is pictured in down panel of Fig.2. Based on proposed in the above mentioned paper identification of the following points $\nu_{(0t_2)} = 30$ Hz; $\nu_{(0t_6)} = 92$ Hz and $\nu_{(0t_{10})} = 150$ Hz we extract parameters ν_0 and h entering in our spectral formulae for torsional mode, equations (26)-(27).

In Fig.3 we highlight by triangles two non-identified before low-frequency QPOs in data for SGR 1806-20 (Israel et al 2005), namely $\nu = 18$ Hz and $\nu = 26$ Hz. The extrapolation to low-frequency region of spectral formulae (13) and (15) leads us to conclude that the latter low-frequency points can be interpreted as manifestation of the dipole toroidal, ${}_0t_1$, and the dipole spheroidal, ${}_0s_1$, overtones of nodeless shear elastic vibrations, respectively.

As to high-frequency points $\nu = 626$ Hz and $\nu = 1840$ Hz in data for SGR 1806-20 is concerned, the lack of observational data makes the above scheme of identification less effective. Putting these points on spectral curve for torsional mode $\nu_{(0t_\ell)}$ extended to very high value of ℓ , we get $\nu_{(0t_{\ell=42})} = 625$ Hz and $\nu_{(0t_{\ell=122})} = 1840$. However, if one puts these points on the spectral curve for spheroidal mode $\nu_{(0s_\ell)}$, we obtain the following identification $\nu_{(0s_{\ell=30})} = 625$ Hz and $\nu_{(0t_{\ell=87})} = 1840$. The last case may be more favorable because the higher multipole degree of oscillations the less their lifetime (Bastrukov et al 2007b), and low- ℓ overtones have, therefore, more chances for surviving. But one must admit that this viewpoint is highly questionable and should be thought of as suggestive, not conclusive.

4 Summary

The exact spectral formulae, which has been obtained here within the framework of Newtonian, non-relativistic, solid-mechanical theory of seismic vibration for the first time, are interesting in its own right from the viewpoint of general theoretical seismology (e.g. Lay, Wallace 1995) because they can be utilized in the study of seismic vibrations of more wider class of solid celestial objects such as Earth-like planets. One of the remarkable findings of our investigation is that the dipole overtones of nodeless elastic shear vibrations trapped in the finite-depth crust of seismically active neutron star possess properties of Goldstone soft modes. It is shown that obtained spectral equations are consistent with the existence treatment of low-frequency QPOs in the X-ray luminosity of flares SGR 1900+14 and

SGR 1806-20 as caused by quake-induced torsional nodeless vibrations (Samuelsson & Andersson 2007a; 2007b). What is newly disclosed here is that previously non-identified low-frequency QPOs in data for SGR 1806-20 can be attributed to nodeless dipole torsional and spheroidal vibrations, namely, $\nu({}_0t_1) = 18$ Hz and $\nu({}_0s_1) = 26$ Hz.

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